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Stable inclusion of *a priori* knowledge of field behaviour in the FDTD algorithm: Application to the analysis of microstrip lines.

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Introduction The FDTD method is well established as a versatile electromagnetic analysis technique [1]. When used to analyse structures such as narrow microstrip lines however, a very small unit cell size is required to accurately characterise the singular nature of the fields and the electrically small strip width. The resultant increase in the number of FDTD unit cells and the corresponding decrease in the algorithm time step [1] create increased demands for both storage and computation in the algorithm.

One technique that potentially enables the treatment of microstrip (and other problematic structures) without recourse to small unit cells is to include *a priori* knowledge of the field behaviour in the algorithm. This contribution presents for the first time a general methodology which accomplishes this task and yet has no adverse effect on the standard algorithm's stability.

Background The fields electrically close to the microstrip edges are dominated by their known static forms and incorporation of this knowledge into FDTD, by modifying the algorithm coefficients, is known to achieve a dramatic improvement in accuracy [2]. These modifications however almost invariably cause instability (an unbounded increase in the solution magnitude) and this renders the technique unusable.

A recent development in the study of FDTD stability has been the use of a passive equivalent circuit for the algorithm [3], this technique, hitherto only used for stabilisation of the FDTD contour path method [4], can also be used to provide an alternative and entirely stable methodology for the inclusion of known field behaviour into the general FDTD algorithm.

The Equivalent Circuit The FDTD algorithm is derived from the replacement of the spatial derivatives in Maxwell's curl equations by centred differences – yielding a continuous-time equation for each field component in the model. Subsequently the time derivatives in the equations are replaced by centred differences to give an explicit discrete-time algorithm. For convenience the first set of equations are referred to here as the *continuous-time* FDTD model and the second set as the *discrete-time* FDTD model.

A full description of the FDTD equivalent circuit is given in [3], it suffices here simply to restate that if a passive circuit can be constructed where the voltage variables obey the same equations as the field variables in the continuous-time FDTD model, the corresponding discrete-time FDTD model must be stable for some finite value of time step.

The result of the modifications to the FDTD algorithm described in [2] was that no passive equivalent to the continuous-time algorithm existed and stability was not guaranteed. If however the inclusion of *a priori* knowledge in FDTD can be made entirely consistent with the existence of an equivalent circuit then stability is assured – this approach is described in the following sections.

Correction Scheme for Metal Strips A section of the FDTD model (unit cell size Δ), complete with a number of the relevant equivalent circuit components, is shown in figure 1. As described in [3] the equivalent circuit consists of capacitors and gyrators (a gyrator is a passive impedance converter with the properties illustrated by the inset in figure 1). As described above, the voltages across the capacitors have the precisely same behaviour as the corresponding fields in the FDTD algorithm and therefore the quantities h_y and e_z refer to either FDTD field components or equivalent circuit voltages, depending on the context.

Consideration of the role played by the various electrical components of the circuit makes it clear that the capacitors represent the mechanism of energy storage associated with each field component and the gyrators represent the means of energy transfer between electric and magnetic components. This observation suggests a scheme for including *a priori* knowledge in the algorithm. The scheme for the calculation of modified values for the components C_1 and G_1 is presented here; the method for the other components proceeds via similar arguments.

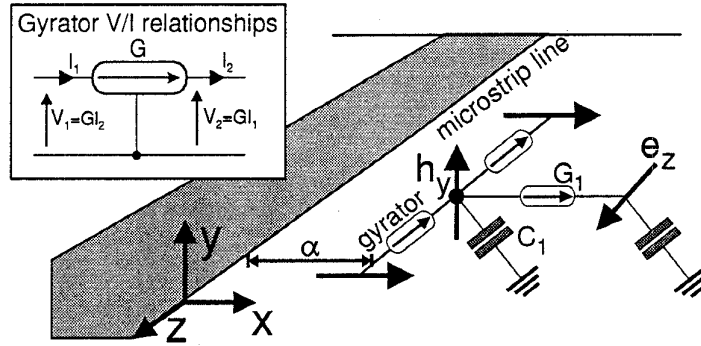


Figure 1: Microstrip line and FDTD mesh (*inset*: gyrator voltage/current relations).

In the equivalent circuit the energy stored in the capacitor C_1 is:

$$\mathcal{E} = \frac{1}{2} C_1 h_y^2 \quad (1)$$

and the power flow $\mathcal{P} = VI$ through G_1 is given by:

$$\mathcal{P} = G_1 e_z h_y \quad (2)$$

Given f_1 and f_2 (two functions describing the known static behaviour of the fields close to the metal edge [5]) with the x, y origin taken to be at the edge of the strip, the field components have the following behaviour close to the edge:

$$H_y(x, y) \propto f_1(x, y) \quad E_z(x, y) \propto f_2(x, y) \quad (3)$$

Let these functions be normalised in order that their values at the positions of the defined FDTD field components are h_y and e_z respectively:

$$H_y(x, y) = \frac{f_1(x, y)}{f_1(\alpha, 0)} h_y \quad E_z(x, y) = \frac{f_2(x, y)}{f_2(\alpha + \frac{\Delta}{2}, 0)} e_z \quad (4)$$

The *a priori* knowledge of the field behaviour yields the power flow and storage in the physical fields. This knowledge can be used to modify the values of capacitors and gyrators in the equivalent circuit, as shown in the following sections.

Capacitor Modification The magnetic energy stored in the $\Delta \times \Delta \times \Delta$ volume of space with h_y at its centre is the volume integral of the energy density $\frac{1}{2}\mu H^2$ [6, p.233], thus:

$$\mathcal{E} = \mu \frac{1}{2} \frac{h_y^2}{f_1^2(\alpha, 0)} \int_{-\Delta}^0 \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \int_{\alpha-\frac{\Delta}{2}}^{\alpha+\frac{\Delta}{2}} f_1^2(x, y) dx dy dz \quad (5)$$

Comparing (5) and (1) gives the value of the capacitor C_1 :

$$C_1 = \mu \frac{1}{f_1^2(\alpha, 0)} \int_{-\Delta}^0 \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} \int_{\alpha-\frac{\Delta}{2}}^{\alpha+\frac{\Delta}{2}} f_1^2(x, y) dx dy dz \quad (6)$$

Gyrator Modification The power flow in an electromagnetic field is given by the integral of the Poynting vector [6, p.465]. Considering the power being transferred across a surface at $x = x_1$ between e_z and h_y :

$$\mathcal{P} = \int_{-\Delta}^0 \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} E_z(x_1, y, z) H_y(x_1, y, z) dy dz \quad (7)$$

A simple approximation to this expression, which was found to give good results, is:

$$\mathcal{P} = \frac{e_z h_y}{f_1(\alpha, 0) f_2(\alpha + \frac{\Delta}{2}, 0)} \int_{-\Delta}^0 \int_{-\frac{\Delta}{2}}^{\frac{\Delta}{2}} f_1(\alpha, y) f_2(\alpha + \frac{\Delta}{2}, y) dy dz \quad (8)$$

Comparison of (8) and (2) gives the appropriate value of G_1 .

The insertion of modified gyrators and capacitors in the equivalent circuit gives rise to a modified FDTD algorithm which incorporates the known field behaviour close to the microstrip edge. The changes to the FDTD algorithm have been limited to alterations to the components of the equivalent circuit and hence the problem of instability will be avoided.

Validation of Correction Scheme In this section a simple boxed microstrip line (width 2 mm, substrate $\epsilon_r = 8.875$, substrate thickness 1 mm) was analysed using the method described above. Results were sought for the effective permittivity of the line as this parameter is known to be very sensitive to modelling accuracy. For comparison the Spectral Domain Method (SDM), in the form described in [7], was employed to analyse the same structure – this method is known to be capable of highly accurate characterisations of microstrip structures.

Figure 2 shows the variation in the effective permittivity of the microstrip against frequency as calculated by SDM, the standard FDTD method and the improved scheme described by this contribution.

It is clear from figure 2 that the equivalent circuit based correction scheme yields a marked improvement in the accuracy of the results, at all frequencies reducing the error to a small fraction of that produced with FDTD alone.

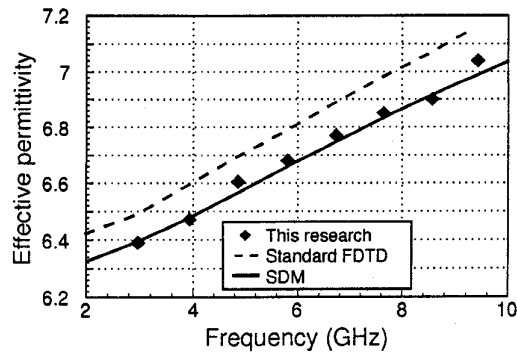


Figure 2: Effective permittivity of microstrip line against frequency.

As described above, the stability of the algorithm is guaranteed. In all cases a choice of time step unchanged from that required by the standard algorithm gave an entirely stable solution (tested up to 60,000 iterations).

Conclusions A general methodology for incorporating *a priori* knowledge into the FDTD algorithm has been presented. In this contribution the technique has yielded a modified FDTD algorithm for the analysis of microstrip; this method has been shown to be both more accurate than the standard FDTD technique and entirely stable.

It is clear that the use of the methodology described herein is far from the only possible application and the general technique can easily be extended to enable the improved treatment of any structure for which *a priori* knowledge of the fields exists.

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